Dr. S.D. Singh (Physics) B.Sc Part I (Hons)
S.B. College, Ara

Q What do you mean by free and force vibration. Obtain an equation for motion of a System under going damped oscillation ?

Ans :- Free vibration :-

When an elastic body is slightly displaced out of its position of stable equilibrium it will in general tend to come back to its original position. But actually this original position is hardly acheived by single reversal of displacement. The body usually over shoots its mean position and then oscillates about it, until the oscillation are stopped by some external force. The time between two successive passage in the same sense through the equilibrium conditions is known as the period of vibration. This body may be made to vibrate at any time and it will always vibrate with same time period, such vibration are called free vibration.

Force Vibration :→

When a body is allowed to vibrate under the influence of external periodic force in the begining of body tries to vibrate with its own natural frequency, but then vibration vanish by soon and the body vibrate with the frequency of applied external force.

This type of vibration is called force vibration. The amplitude of the force vibration depends on the relation between the frequency of the applied force and the natural frequency of body

When a A.C is passed through a stretched string between the poles of magnet. the string vibrate with frequency of A.C. This vibration is called force vibration.

Equation of motion under damped oscillations:→

when a body having mass 'm' vibrates under a medium which exerts damping force on the body. The eq^n of motion is modified. The resisting force is proportional to the velocity we can write $b\frac{dx}{dt}$; 'b' is damping constant. The force acts against the inertial force $m\frac{d^2x}{dt^2}$ due to its motion over and above the resisting force.

Equation of motion may be written as

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - ax \quad \text{—— (1)}$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{a}{m}x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + 2K\frac{dx}{dt} + \omega^2 x = 0 \quad \text{—— (2)}$$

where $2K = \frac{b}{m}$ = The resisting force per unit mass per unit velocity & $\omega^2 = \frac{a}{m}$ = The restoring force per unit mass per unit displacement

Let the treal sol^n of (1) be $x = Ae^{mt}$

$$\frac{dx}{dt} = mAe^{mt} \quad \& \quad \frac{d^2x}{dt^2} = m^2Ae^{mt}$$

from eq^n (1)

$$Ae^{mt}(m^2 + 2Km + \omega^2) = 0$$

$$\Rightarrow m^2 + 2Km + \omega^2 = 0$$

$$\therefore m = -K \pm \sqrt{K^2 - \omega^2}$$

The eq^n (1) becomes

$$x = A_1 e^{(-K+\sqrt{K^2-\omega^2})t} + A_2 e^{(-K-\sqrt{K^2-\omega^2})t} \quad \text{—— (3)}$$

Depending on the condition of damping. This

Solution takes three forms

(I) If the damping is very large

i.e if $K > \omega$, $\sqrt{K^2-\omega^2}$ is positive

Here the displacement x has two terms both diminishing exponentially and no vibration is excuted. This is called dead beat or over damped motion. it is found in a dead beat moving coil galvanometers.

(II) when $K = \omega$

The solution is $x = (A_1 + A_2) e^{-Kt} = ce^{-\frac{b}{2m} \cdot t}$

This represents a non oscillatory motion which is critically damped.

(III) when $K < \omega$, $\sqrt{K^2-\omega^2}$ is imaginary

Let $\sqrt{K^2-\omega^2}$ be written as $i\beta$ where $i = \sqrt{-1}$

& $\beta^2 = \omega^2 - K^2$

$$\therefore \; x = A_1 e^{-Kt+i\beta t} + A_2 e^{-Kt-i\beta t}$$

$$= e^{-Kt}(A_1 e^{i\beta t} + A_2 e^{-i\beta t})$$

$$= c_1 e^{-Kt} \sin(\beta t + \delta)$$

Here $c_1 = \delta =$ new constant

The equation evidently represents a damped harmonic motion. The amplitude of the vibration is ce^{-Kt} which dies off exponentially with time. The natural frequency of the damped vibration is here $\beta/2\pi$ instead of $\frac{\omega}{2\pi}$. The latter being the natural frequency of an undamped free vibration. Since $\beta = \sqrt{\omega^2-K^2}$ & K is generally very small, numerically β & ω are nearly identical for most of the cases.